

## 第6回代数・解析・幾何学セミナー

表記のセミナーを下記の要領で開催致しますのでご案内申し上げます。

日 時：2011年2月14日（月）～17日（木）

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なお、本セミナーは、下記の支援を受けています。

平成22年度広島大学学長裁量経費

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### プログラム

2月14日（月）

13:30～14:20 : Ken Saito (Osaka City University)

“Les livres arithmétiques d’Euclide et l’induction complète”

14:30～15:20 : Yoshiki Oshima (Tokyo University)

“Discrete branching laws of derived functor modules”

15:40～16:30 : Satoshi Aoki (Kagoshima University)

“Gröbner bases theory in designed experiments”

16:40～17:30 : Shin-ya Koyama (Toyo University)

“Absolute zeta functions”

2月15日（火）

9:30～10:20 : Toshiyuki Akita (Hokkaido University)

“Mackey functors associated with surface symmetries”

10:30～11:20 : Tadakazu Sawada (Tohoku University)

“Frobenius sandwich singularities and surfaces”

11:30～12:20 : Masahiko Yoshinaga (Kyoto University)

“Minimal Stratifications for Line Arrangements”

14:00～14:50 : Jörg Schürmann (Universität Münster)

“Generating series for invariants of symmetric products”

15:00～15:50 : Takuro Abe (Kyoto University)

“Chambers and freeness of line arrangements”

16:10～17:00 : Pierre Schapira (Université Paris VI)

“Sheaf quantization of Hamiltonian isotopies and applications”

18:30～ Dinner Party

2月16日(水)

9:30~10:20 : Tatsuo Suwa (Hokkaido University)

“Bott–Chern revisited”

10:40~11:30 : Hiroyuki Nakaoka (Kagoshima University)

“On a generalization of the Dress construction for Tambara functors,  
and its application to the Witt–Burnside construction”

11:40~12:30 : Go Yamashita (Toyota Central RD Labs., Inc.)

“Pure maths and applied maths—from a personal point of view—”

14:00~14:50 : Osamu Saeki (Kyushu University)

“Topology of definite fold singularities”

15:20~16:30 : Kazushi Ueda (Osaka University)

“Dimer models and triangulated categories”

16:40~17:30 : Jinsung Park (KIAS)

“The Chern–Simons invariant for Schottky manifolds”

2月17日(木)

9:30~10:20 : Tohsuke Urabe (Ibaraki University)

“New ideas for resolution of singularities in arbitrary characteristic”

10:40~11:30 : Jiro Sekiguchi (Tokyo University of Agriculture and Technology)

“Saito free divisors and holonomic systems”

11:40~12:30 : Markus Banagl (Universität Heidelberg)

“Is there a cohomology theory stable under smooth deformation of singularities?”

(\*) 2月15日(火)の懇親会に出席予定の方は、人数把握のため、遅くとも2月8日(火)までに與倉までメールでお知らせ下さい。

## Abstracts of Talks

- Takuro Abe (Kyoto University)

Title: “Chambers and freeness of line arrangements”

Abstract: Connected components of a real arrangement are called chambers. We give a lower bound of chambers of line arrangements under certain conditions. Also we will show an interesting relation between counting chambers and some algebraic aspects of arrangements, called freeness.

- Toshiyuki Akita (Hokkaido University)

Title: “Mackey functors associated with surface symmetries”

By a surface symmetry we mean a pair  $(G; C)$ , where  $C$  is a closed oriented (real) surface and  $G$  is a finite group acting on it. In this talk, we will introduce Mackey functors associated with surface symmetries. Equivariant characteristic classes such as Mumford–Morita–Miller classes and “local part” of homology representations can be considered as natural transformations of Mackey functors. In addition, we will discuss Riemann–Roch type formulae for these natural transformations.

- Satoshi Aoki (Kagoshima University)

Title: “Gröbner bases theory in designed experiments”

Abstract: Application of Groebner bases theory to designed experiments is an attractive topic in a relatively new field in statistics, called computational algebraic statistics. In this talk, we revisit some fundamental results in this field and some works by Aoki and Takemura, mainly in the treatments of two-level fractional factorial designs.

- Markus Banagl (Universität Heidelberg)

Title: “Is there a cohomology theory stable under smooth deformation of singularities?”

Abstract: In many situations, it is homotopy theoretically possible to associate to a singular space in a natural way a generalized geometric Poincaré complex, whose cohomology turns out to be a new cohomology theory for singular spaces, not isomorphic in general to intersection cohomology or  $L^2$ -cohomology. An alternative description of the new theory by a de Rham complex of global differential forms is available. The theory has a richer internal algebraic structure than intersection cohomology and also addresses questions in type II string theory. While intersection cohomology is stable under small resolutions, the new theory is often stable under deformations of singularities. The latter result is partly joint work with Laurentiu Maxim.

- Shin-ya Koyama (Toyo University)

Title: “Absolute zeta functions”

Abstract: This is a survey talk on absolute mathematics and absolute zeta functions. We introduce the absolute zeta functions of Weil type, and show that they satisfy an additive structure of zeros and poles. This is a joint work with N. Kurokawa (Tokyo Institute of Technology) and S. Kim (KAIST).

- Hiroyuki Nakaoka (Kagoshima University)

Title: “On a generalization of the Dress construction for Tambara functors, and its application to the Witt–Burnside construction”

Abstract: For a finite group  $G$ , Mackey functors and Tambara functors are regarded as  $G$ -bivariant analogs of abelian groups and commutative rings. As such, many naive algebraic properties concerning rings and groups have been extended to these  $G$ -bivariant analogous notions. In this talk, we introduce a  $G$ -bivariant analog of the group-ring construction. It generalizes the Dress construction, and also has some relation to the Witt–Burnside construction. As a byproduct, we also obtain a  $G$ -bivariant analog of the polynomial ring.

- Yoshiki Oshima (Tokyo University)

Title: “Discrete branching laws of derived functor modules”

Abstract: Zuckerman functor provides a certain class of unitary representations of real reductive Lie groups, known as  $A_{\mathfrak{q}}(\lambda)$ -modules. In this talk, we study branching laws of  $A_{\mathfrak{q}}(\lambda)$  for reductive symmetric pairs. When  $A_{\mathfrak{q}}(\lambda)$  is discretely decomposable, we obtain explicit branching formulas by using  $D$ -module theory.

- Jinsung Park (KIAS)

Title: “The Chern–Simons invariant for Schottky manifolds”

Abstract: In this talk we will introduce the Chern–Simons invariant for Schottky 3-manifolds. The Schottky manifold is a non-compact hyperbolic manifold with infinite volume, whose boundary is a Riemann surface. We will explain that this Chern–Simons invariant defines a natural complex valued function over deformation space of the hyperbolic structures if we combine it with the renormalized volume of the Schottky manifold. We will also explain some basic properties of this invariant and related open questions.

- Osamu Saeki (Kyushu University)

Title: “Topology of definite fold singularities”

Abstract: It is known as the Reeb theorem that if a closed differentiable manifold admits a smooth function with only minima and maxima as its critical points, then the manifold is necessarily homeomorphic to the sphere. In this talk some generalizations of this theorem will be presented for smooth maps into higher dimensional spaces. Unlike the function case, the existence of such maps strongly affects the differentiable structure of the manifold. In the second part, elimination of definite fold singularities will be discussed in relation to broken Lefschetz fibrations.

- Ken Saito (Osaka City University)

Title: “Les livres arithmétiques d’Euclide et l’induction complète”

Abstract: Mathematical induction in history of mathematics:

Who was the first to use mathematical induction (complete induction), and did Greeks use mathematical induction? This was the theme of a polemic among historians of Greek mathematics in 1990’s, which has yielded no definitive conclusion.

It is true that some arguments do exist which can be interpreted as the use of mathematical induction. However, in the corpus of Greek mathematica works, fairly complicated propositions, which could have been proved much more simply and effectively by induction, were often proved by ‘quasi-general’ argument. This implies that mathematical induction was not recognized as an effective demonstrative technique.

Sporadic use induction can also be found in Levi Ben Gershon (14C) and Francesco Maurolico (16C), but conscious formulation of inductive argument is due to Pascal, and it is still later that induction developed into an effective demonstrative technique.

Therefore, “who first used mathematical induction?” or “Did Greeks use mathematical induction?” are typically wrong questions in history of mathematics. The answer would be “inductive thinking did exist as is attested in sorites paradox, but there was no recognition in ancient Greece that induction could be effective technique in mathematical proof.”

- Tadakazu Sawada (Tohoku University)

Title: “Frobenius sandwich singularities and surfaces”

Abstract: Let  $X$  be a smooth variety over an algebraically closed field of positive characteristic. A Frobenius sandwich of  $X$  is a normal variety through which the Frobenius morphism of  $X$  factors. It is natural to ask what kinds of singularities and varieties appear as Frobenius sandwiches. We will discuss Frobenius sandwiches locally and globally from the viewpoint of Frobenius splitting and present classifications of the globally  $F$ -regular Frobenius sandwiches of the projective plane and Hirzebruch surfaces. This is a joint work with N. Hara.

- Pierre Schapira (Université Paris VI)

Title: “Sheaf quantization of Hamiltonian isotopies and applications”

Abstract: Recently Tamarkin presented a new approach to symplectic topology based on the microlocal theory of sheaves. For that purpose he had to adapt this theory which relies on the homogeneous symplectic structure to the non homogeneous case. Here, we remain in the homogeneous symplectic setting and prove various results of non displaceability, including the conservation of Morse inequalities as well as some results specific to positive isotopies. The main tool is a theorem which asserts that any Hamiltonian isotopy admits a unique sheaf quantization.

- Jörg Schürmann (Universität Münster)

Title: “Generating series for invariants of symmetric products”

Abstract: We explain new formulae for the generating series of Hodge theoretical invariants for symmetric products of complex quasi-projective varieties and mixed Hodge module complexes. These invariants include the corresponding Hodge polynomial as well as Hirzebruch characteristic classes, including those associated to middle intersection cohomology. This is joint work with L. Maxim, M. Saito, S. Cappell, J. Shaneson and S. Yokura.

- Jiro Sekiguchi (Tokyo University of Agriculture and Technology)

Title: “Saito free divisors and holonomic systems”

Abstract: The purpose of this talk is to report many examples of holonomic systems with regular singularities along Saito free divisors. Typical example of Saito free divisors is the zero set of an irreducible real reflection group. The speaker focuses his attention on such divisors in  $\mathbb{C}^3$ . In particular he explains the definition of Saito free divisors and discuss on the holonomic systems in question.

- Tatsuo Suwa (Hokkaido University)

Title: “Bott–Chern revisited”

Abstract: In a paper published in 1965, R. Bott and S.S. Chern gave an explicit formula involving the top Chern form of a Hermitian vector bundle. This is used to prove the relative Gauss–Bonnet theorem, which in turn is applied to produce generalizations of various Nevanlinna type theorems on value distribution. In this talk we take up these results on Chern forms from the viewpoint of analytic intersection theory based on residues and give some generalizations.

- Kazushi Ueda (Osaka University)

Title: “Dimer models and triangulated categories”

Abstract: A dimer model is a bicolored graph on a real 2-torus which encodes the information of an A-infinity category. This A-infinity structure is cyclic of dimension 3, which implies that the Serre functor on the derived category of this A-infinity category is a 3-shift functor. In the talk, I will discuss my joint work with Akira Ishii and Masahiro Futaki on the relation between dimer models and homological mirror symmetry for toric Calabi–Yau 3-folds.

- Tohsuke Urabe (Ibaraki University)

Title: “New ideas for resolution of singularities in arbitrary characteristic”

Abstract: Recently I succeeded in showing “local uniformization theorem in arbitrary characteristic and in arbitrary dimension”. See my manuscript “New Ideas for Resolution of Singularities in Arbitrary Characteristic” (<http://arxiv.org/abs/1011.1083>) There exists a formulation of the local theory of resolution of singularities due to Zariski applying valuation theory. This formulation is called “local uniformization theorem”. “ocal uniformization theorem in characteristic zero” is Zariski’s result and it appeared in 1940. “local uniformization theorem in arbitrary characteristic in dimension less than or equal to two” is Abhyankar’s result and its essential part appeared in 1956. Since then for long “local uniformization theorem in arbitrary characteristic and in arbitrary dimension” had been an open problem. The chief ingredient in my verification is the toric theory. We add a lot of exact theory of convex sets to the toric theory and apply it.

After referring to the Newton polyhedron of the singularity to be resolved, we construct an effective composition of blowing-ups. Then we can show that the singularity is improved by the composition of blowing-ups. This is very different from Zariski’s theory and Hironaka’s theory based on the idea of the maximal contact. The big problem remaining for us is to establish global theory of resolution of singularities in arbitrary characteristic and in arbitrary dimension.

- Go Yamashita (Toyota Central RD Labs., Inc.)

Title: “Pure maths and applied maths—from a personal point of view—”

Abstract: The speaker is working on the pure maths research (especially arithmetic geometry) in a private company Toyota Central RD Labs., Inc from the last July.

It was exciting experience for him to communicate with applied mathematicians, physicists, and engineers, and to know how the pure maths is applied. Through these experiences and the discussions on “conflicts of pure maths research and applied maths research” with his colleagues, he was also a little bit involved with deep thinking on the social matters....

The following topics, which one of the organizers asked him to talk about from his point of view in his company, will be contained:

- “the relationship between pure and applied maths”,
- “the social tasks of Japanese pure maths society”,
- “the problems in Japanese pure maths society for future”.

In the first 10 minutes, he will also talk about the recent joint work on  $R = T$  with S. Yasuda (RIMS).  $R = T$  was the technique founded by Taylor–Wiles for the proof of Fermat conjecture. Roughly speaking, their joint work (based on the speaker’s previous work) is a mixture of

- Kisin’s technique, by which he weakened the condition at  $p$  for  $GL(2)$  in Taylor–Wiles’ works on the modularity lifting (by using his theory, Serre’s conjecture was proved), and
- Clozel–Harris–Taylor, and Taylor, by which they generalised Taylor–Wiles’ works for  $GL(n)$  (by using their theory, Sato–Tate conjecture was proved).

- Masahiko Yoshinaga (Kyoto University)

Title: “Minimal stratification for line arrangements”

Abstract: Several types of minimal presentations of  $\pi_1$  for arrangements complements have been known since long time. More generally, arrangements complements are known to be homotopy equivalent to minimal  $CW$  complex. In this talk, we focus on two dimensional complexified real arrangements and introduce “minimal stratification” which can be considered as a dual notion to minimal  $CW$  complex.