

Gerrit van Dijk (Leiden University): ‘Gelfand pairs, old and new’

**Abstract.**

Let  $G$  be a Lie group with a bi-invariant positive measure and let  $H$  be a closed subgroup of  $G$  satisfying the same conditions. Set  $S=G/H$ . Let  $D(S)$  be the space of test functions on  $S$  and let  $D'(S)$  be its continuous anti-dual. The group  $G$  acts on both spaces by left translations. Let  $R$  be a continuous unitary irreducible representation of  $G$  on a Hilbert space  $V$ .

We shall discuss the following two problems:

1. Can  $R$  be realized on  $D'(S)$ , i.e. does there exist a continuous linear injection from  $V$  to  $D'(S)$ , commuting with the action of  $G$ ?
2. If such a realization exists, is it unique (up to scaling)?

We will focus on Problem 2 (which is related to the classical theory of Gelfand pairs). We shall show recent answers and indicate an interesting link with number theory.