Toric duality for deformations of the cyclic quotient surface singularities $A_{n,q}$ and $A_{n,n-q}$

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Cyclic quotient surface singularities are completely determined by relatively prime numbers n, q with $1 \leq q < n$ such that we can refer to them as the singularities $A_{n,q}$.
They are also the normal affine two-dimensional *toric varieties*. As such, the singularities $A_{n,q}$ and $A_{n,n-q}$ are *dual* to each other in the sense that the underlying *strongly convex rational cones* are duals of each other.
It is well known that the HIRZEBRUCH-JUNG continued fraction expansion for n/q encodes the geometry of the *minimal resolution* of the corresponding singularity $A_{n,q}$, whereas the expansion for $n/n-q$ encodes the *equations* of the singularity.
The singularities $A_{n,q}$ possess a rich deformation theory. In general, the total space of the (reduced) *versal deformation* admits many (normal) components. After a finite covering - which kills *monodromy* - these components have also a *toric structure*.
Among them, in particular, there is the ARTIN component which one obtains by blowing down the versal deformation of the minimal resolution of $A_{n,q}$.
It is the purpose of this quite elementary lecture to explain MARTIN HAMM's striking observation in his Hamburg dissertation that these toric structures of the ARTIN components of $A_{n,q}$ and $A_{n,n-q}$ are again dual to each other. In other, more concrete terms: The HIRZEBRUCH-JUNG continued fraction expansion for $n/n-q$ encodes the *generators* defining the algebra of the ARTIN component whereas the expansion for n/q encodes the *inequalities* defining the corresponding strongly convex rational cone.