

平成 29 年度
鹿児島大学大学院理工学研究科入学試験
博士前期課程 数理情報科学専攻
英語

平成 28 年 8 月 18 日 10:00 - 11:30

注意

- (1) 配布物は、問題冊子 (A4, 3 枚), 解答用紙 (B4, 2 枚), 草案用紙 (B4, 2 枚) である.
- (2) 試験開始の合図があるまで、問題冊子を開いてはならない.
- (3) 出題数は **1**, **2** の 2 題で、2 題とも解答せよ.
- (4) 試験開始後、すべての解答用紙に受験番号を記入せよ.
- (5) 解答用紙が不足する場合には裏面を使用してもよい.
- (6) 英和辞書を使用してもよいが、電子辞書の使用は認めない.
- (7) 問題冊子と草案用紙は持ち帰ること.

1 以下の英文の全文を和訳せよ.

To every compact Riemannian manifold M there corresponds the sequence $0 = \lambda_1 < \lambda_2 < \lambda_3 < \dots$ of eigenvalues for the Laplace operator on M . It is not known just how much information about M can be extracted from this sequence. This note will show that the sequence does not characterize M completely, by exhibiting two 16-dimensional toruses which are distinct as Riemannian manifolds but have the same sequence of eigenvalues.

出典 Milnor, J. Eigenvalues of the Laplace operator on certain manifolds. Proc. Nat. Acad. Sci. U.S.A. **51** (1964) 542.

参考 Riemannian manifold : リーマン多様体

2 以下の英文はミルザハニが 2014 年の国際数学会議においてフィールズ賞を受賞した際の Curtis T. McMullen による業績紹介 “The work of Maryam Mirzakhani” の抜粋である。全文を和訳せよ。

Mirzakhani has established a suite of powerful new results on orbit closures and invariant measures for dynamical systems on moduli spaces. She has also given a new proof of Witten’s conjecture, which emerges naturally from a counting problem for simple closed geodesics on Riemann surfaces. This note gives a brief discussion of her main results and their ramifications, including the striking parallels between homogeneous spaces and moduli spaces that they suggest.

中略

Moduli space can be presented as the quotient $\mathcal{M}_g = \mathcal{T}_g/\text{Mod}_g$ of Teichmüller space – its universal cover, a contractible bounded domain in \mathbb{C}^{3g-3} – by the action of the mapping-class group of a surface. One of the challenges of working with moduli space is that it is totally inhomogeneous: for example, the symmetry group of \mathcal{T}_g (as a complex manifold) is simply the discrete group Mod_g (for $g > 2$). One of Mirzakhani’s remarkable contributions is to show that, nevertheless, dynamics on moduli space displays many of the same rigidity properties as dynamics on homogeneous spaces.

参考 moduli spaces : モジュライ空間