平成 28 年度
鹿児島大学大学院理工学研究科入学試験
博士前期課程　数理情報科学専攻

英語

平成 27 年 8 月 18 日 (火)　10:00–11:30

注意.

1. 配布物は, 問題冊子 (A4, 3 枚), 解答用紙 (B4, 2 枚), 草案用紙 (B4, 2 枚) である.
2. 試験開始の合図があるまで, 問題冊子を開いてはならない.
3. 出題数は **1**, **2** の 2 題で, 2 題とも解答せよ.
4. 試験開始後, すべての解答用紙に受験番号を記入せよ.
5. 解答用紙が不足する場合には裏面を使用してもよい.
6. 英和辞書を使用してもよいが, 電子辞書の使用は認めない.
7. 問題冊子と草案用紙は持ち帰ること.

1 以下の英文の全文を和訳せよ.

We will suppose that all vector spaces are real. Let E be a vector space. A mapping $\|\cdot\| : E \to \mathbb{R}$ is said to be a *norm* if, for all $x, y \in E$ and $\lambda \in \mathbb{R}$ we have

- $\|x\| \geq 0$;
- $\|x\| = 0 \iff x = 0$;
- $\|\lambda x\| = |\lambda|\ \|x\|$;
- $\|x + y\| \leq \|x\| + \|y\|$.

The pair $(E, \|\cdot\|)$ is called a *normed vector space* and we say that $\|x\|$ is the norm of x.

中略

When there is no confusion, we will simply write E for a normed vector space. To distinguish norms on different normed vector spaces we will often use suffixes. For example, if we are dealing with the normed vector spaces E and F, we may write $\|\cdot\|_E$ for the norm on E and $\|\cdot\|_F$ for the norm on F. The most common norms on $\mathbb{R}^n$ are defined as follows:

$$\|x\|_1 = |x_1| + \cdots + |x_n|, \quad \|x\|_2 = \sqrt{x_1^2 + \cdots + x_n^2} \qquad \text{and}$$

$$\|x\|_\infty = \max\{|x_1|, \cdots, |x_n|\},$$

where x_i is the ith coordinate of x.

出典 R Coleman, “Calculus on Normed Vector Spaces,” Universitext, Springer, 2012.

参考 $\mathbb{R}$: 実数全体の集合, norm: ノルム, normed vector spaces: ノルム空間,
$\mathbb{R}^n$: n 次元数ベクトル空間

2 以下の英文の全文を和訳せよ.

Definition 3.17 *A map* $f : X \to Y$ *is said to be* invertible *if there exists a map* $g : Y \to X$ *such that the composition* $g \circ f$ *is the identity map of* X *and the composition* $f \circ g$ *is the identity map of* Y.

We immediately get a criterion on f for it to be invertible:

Proposition 3.18 *A map* $f : X \to Y$ *is invertible if and only if it is bijective.*

Proof Suppose first that f is invertible and let g be as in Definition 3.17. Then

$$f(x) = f(x') \Rightarrow g\left(f(x)\right) = g\left(f(x')\right) \Rightarrow x = x'$$

so f is injective. Also, given any $y \in Y$ we have $y = f\left(g(y)\right)$ so $y \in f(X)$, which says that f is onto. Hence f is bijective.

Secondly suppose that f is bijective. We may define $g : Y \to X$ as follows: for any $y \in Y$ we know f is onto, so $y = f(x)$ for some $x \in X$. Moreover this x is unique for a given y since f is injective. Put $g(y) = x$, and we can see that f and g satisfy Definition 3.17, so f is invertible as required. □

The last part of the above proof also proves

Proposition 3.19 *When* f *is invertible, there is a* unique g *satisfying Definition 3.17. This unique* g *is called* the inverse of f, *written* f^{-1}.

出典 Wilson A. Sutherland, “Introduction to Metric & Topological Spaces.“ Second Edition, Oxford University Press, 2009.

参考 invertible: 可逆