平成27年度
鹿児島大学大学院理工学研究科入学試験
博士前期課程　数理情報科学専攻

英語

平成26年8月19日(火)　10:00–11:30

注意.

1. 配布物は, 問題冊子 (A4, 3枚), 解答用紙 (B4, 2枚), 草案用紙 (B4, 2枚) である.
2. 試験開始の合図があるまで, 問題冊子を開いてはならない.
3. 出題数は **1**, **2** の2題で, 2題とも解答せよ.
4. 試験開始後, すべての解答用紙に受験番号を記入せよ.
5. 解答用紙が不足する場合には裏面を使用してもよい.
6. 英和辞書を使用してもよいが, 電子辞書の使用は認めない.
7. 問題冊子と草案用紙は持ち帰ること.

1 以下の英文の全文を和訳せよ.

An inner product $\langle\,,\,\rangle$ on a real vector space V is said to be *positive definite* if and only if $\langle v, v\rangle > 0$ for all nonzero v in V. The Euclidean inner product on $\mathbb{R}^n$ is an example of a positive definite inner product.

Let $\langle\,,\,\rangle$ be a positive definite inner product on V. The *norm* of v in V, with respect to $\langle\,,\,\rangle$, is defined to be the real number

$$\|v\| = \langle v, v\rangle^{\frac{1}{2}}. \tag{1.3.2}$$

The norm of x in $\mathbb{R}^n$, with respect to the Euclidean inner product, is called the *Euclidean norm* and is denoted by $|x|$.

Theorem 1.3.1. (Cauchy's inequality) *Let $\langle\,,\,\rangle$ be a positive definite inner product on a real vector space V. If v, w are vectors in V, then*

$$|\langle v, w\rangle| \leq \|v\|\|w\|$$

with equality if and only if v and w are linearly dependent.

出典：John G. Ratcliffe, "Foundations of Hyperbolic Manifolds." Second Edition. Graduate Texts in Mathematics 149. Springer, New York, 2006.

参考：Euclidean inner product ユークリッド内積, positive definite 正定値, Euclidean norm ユークリッドノルム, Cauchy コーシー, linearly dependent 線型従属

2 以下の英文の全文を和訳せよ.

Toward the end of the nineteenth century it became clear to many mathematicians that the Riemann integral (about which one learns in calculus courses) should be replaced by some other type of integral, more general and more flexible, better suited for dealing with limit processes. Among the attempts made in this direction, the most notable ones were due to Jordan, Borel, W. H. Young, and Lebesgue. It was Lebesgue's construction which turned out to be the most successful.

In brief outline, here is the main idea: The Riemann integral of a function f over an interval $[a,\, b]$ can be approximated by sums of the form

$$\sum_{i=1}^{n} f(t_i)m(E_i)$$

where $E_1, \ldots, E_n$ are disjoint intervals whose union is $[a,\, b]$, $m(E_i)$ denotes the length of E_i, and $t_i \in E_i$ for $i = 1, \ldots, n$. Lebesgue discovered that a completely satisfactory theory of integration results if the sets E_i in the above sum are allowed to belong to a larger class of subsets of the line, the so-called "measurable sets," and if the class of functions under consideration is enlarged to what he called "measurable functions."

出典：Walter Rudin, "REAL AND COMPLEX ANALYSIS." Third Edition. McGraw-Hill Book Co., New York, 1987.

参考： calculus courses 微積分のコース, Jordan, Borel, W. H. Young, 及び Lebesgue はそれぞれ数学者の名前, Riemann integral リーマン積分, disjoint 互いに交わりを持たない, intervals 区間, measurable sets 可測集合, measurable functions 可測関数