平成25年度
鹿児島大学大学院理工学研究科入学試験
博士前期課程　数理情報科学専攻

英語

平成25年2月13日(水)　10:00-11:30

注意.

1. 配布物は, 問題冊子 (A4, 3 枚), 解答用紙 (B4, 2 枚), 草案用紙 (B4, 2 枚) である.
2. 試験開始の合図があるまで, 問題冊子を開いてはならない.
3. 出題数は[1], [2] の2題で, 2題とも解答せよ.
4. 試験開始後, すべての解答用紙に受験番号を記入せよ.
5. 解答用紙が不足する場合には裏面を使用してもよい.
6. 英和辞書を使用してもよいが, 電子辞書の使用は認めない.
7. 問題冊子と草案用紙は持ち帰ること.

1 以下の英文の全文を和訳せよ.

First let us explain some of our terms. $\mathbb{R}^k$ denotes the k-dimensional euclidean space; thus a point $x \in \mathbb{R}^k$ is an k-tuple $x = (x_1, \cdots, x_k)$ of real numbers.

Let $U \subset \mathbb{R}^k$ and $V \subset \mathbb{R}^l$ be open sets. A mapping f from U to V (written $f\colon U \to V$) is called *smooth* if all of the partial derivatives $\partial^n f / \partial x_{i_1} \cdots \partial x_{i_n}$ exist and are continuous.

More generally let $X \subset \mathbb{R}^k$ and $Y \subset \mathbb{R}^l$ be arbitrary subsets of euclidean spaces. A map $f\colon X \to Y$ is called *smooth* if for each $x \in X$ there exist an open set $U \subset \mathbb{R}^k$ containing x and a smooth mapping $F\colon U \to \mathbb{R}^l$ that coincides with f throughout $U \cap X$.

If $f\colon X \to Y$ and $g\colon Y \to Z$ are smooth, note that the composition $g \circ f\colon X \to Z$ is also smooth. The identity map of any set X is automatically smooth.

DEFINITION. A map $f\colon X \to Y$ is called a *diffeomorphism* if f carries X homeomorphically onto Y and if both f and f^{-1} are smooth.

We can now indicate roughly what *differential topology* is about by saying that it studies those properties of a set $X \subset \mathbb{R}^k$ which are invariant under diffeomorphism.

出典：John W. Milnor, "Topology from the differentiable viewpoint." Princeton Landmarks in Mathematics. Princeton University Press, Princeton, NJ, 1997.

参考：smooth 滑らかな，　diffeomorphism 微分同相，微分同相写像，　homeomorphically 同相的に，同相となるように，　differential topology 微分位相幾何，微分トポロジー

2 以下の英文の全文を和訳せよ.

A *graph* X consists of a *vertex* set $V(X)$ and an *edge* set $E(X)$, where an edge is an unordered pair of distinct vertices of X. We will usually use xy rather than $\{x, y\}$ to denote an edge. If xy is an edge, then we say that x and y are *adjacent* or that y is a *neighbour* of x, and denote this by writing $x \sim y$. A vertex is *incident* with an edge if it is one of the two vertices of the edge. Graphs are frequently used to model a binary relationship between the objects in some domain, for example, the vertex set may represent computers in a network, with adjacent vertices representing pairs of computers that are physically linked.

Two graphs X and Y are equal if and only if they have the same vertex set and the same edge set. Although this is a perfectly reasonable definition, for most purposes the model of a relationship is not essentially changed if Y is obtained from X just by renaming the vertex set. This motivates the following definition: Two graphs X and Y are *isomorphic* if there is a bijection, φ say, from $V(X)$ to $V(Y)$ such that $x \sim y$ in X if and only if $\varphi(x) \sim \varphi(y)$ in Y. We say that φ is an *isomorphism* from X to Y. Since φ is a bijection, it has an inverse, which is an isomorphism from Y to X. If X and Y are isomorphic, then we write $X \cong Y$. It is normally appropriate to treat isomorphic graphs as if they were equal.

出典：Chris Godsil, Gordon Royle, "Algebraic graph theory." Graduate Texts in Mathematics, 207. Springer-Verlag, New York, 2001.

参考： graph グラフ, vertex 頂点, edge 辺, adjacent 隣接する, neighbour 隣接点, incident 接続している