平成25年度
鹿児島大学大学院理工学研究科入学試験
博士前期課程　数理情報科学専攻

英語

平成24年8月21日(火)　10:00-11:30

注意.

1. 配布物は, 問題冊子 (A4, 3 枚), 解答用紙 (B4, 2 枚), 草案用紙 (B4, 2 枚) である.
2. 試験開始の合図があるまで, 問題冊子を開いてはならない.
3. 出題数は $\boxed{1}$, $\boxed{2}$ の 2 題で, 2 題とも解答せよ.
4. 試験開始後, すべての解答用紙に受験番号を記入せよ.
5. 解答用紙が不足する場合には裏面を使用してもよい.
6. 英和辞書を使用してもよいが, 電子辞書の使用は認めない.
7. 問題冊子と草案用紙は持ち帰ること.

$\boxed{1}$ 以下の英文の全文を和訳せよ.

The notions of convergent sequences and Cauchy sequences are obviously meaningful in any metric space. Indeed, we would say that $x_n \to x$ if $d(x_n, x) \to 0$, and we would say that $\{x_n\}$ is a Cauchy sequence if $d(x_n, x_m) \to 0$ as n and m tend to ∞. It is clear that every convergent sequence is a Cauchy sequence. For $\mathbb{R}$ and $\mathbb{C}$ we have proved the converse, namely that every Cauchy sequence is convergent (Chap. 2, Sec. 2.1), and it is not hard to see that this property carries over to any $\mathbb{R}^n$. In view of its importance the property deserves a special name.

Definition 5. *A metric space is said to be complete if every Cauchy sequence is convergent.*

A subset is complete if it is complete when regarded as a subspace. The reader will find no difficulty in proving that *a complete subset of a metric space is closed*, and that *a closed subset of a complete space is complete.*

We shall now introduce the stronger concept of *compactness*. It is stronger than completeness in the sense that every compact space or set is complete, but not conversely. As a matter of fact it will turn out that the compact subsets of $\mathbb{R}$ and $\mathbb{C}$ are the closed bounded sets.

出典：L.V. Ahlfors, "Complex analysis. An introduction to the theory of analytic functions of one complex variable." Third edition. International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., New York, 1978.

参考：Cauchy sequence コーシー列, Chap. 2, Sec. 2.1 第2章節2.1, Definition 5 定義5, complete 完備, compact コンパクト

2 以下の英文の全文を和訳せよ.

Consider now the effect of one rotation followed by another. Suppose that we transform (x, y) into (x', y') by a rotation through ϑ, then (x', y') into (x'', y'') by a rotation through φ. Then we have

$$\begin{aligned} \begin{bmatrix} x'' \\ y'' \end{bmatrix} &= \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \\ &= \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \end{aligned}$$

This suggests that the effect of one rotation followed by another can be described by the product of the corresponding rotation matrices. Now it is intuitively clear that the order in which we perform the rotations does not matter, the final frame of reference being the same whether we first rotate through ϑ and then through φ or whether we rotate first through φ and then through ϑ. Intuitively, therefore, we can assert that *rotation matrices commute*. That this is indeed the case follows from the identities

$$R_\vartheta R_\varphi = R_{\vartheta+\varphi} = R_\varphi R_\vartheta$$

which the reader can easily verify as an exercise using the standard identities for $\cos(\vartheta + \varphi)$ and $\sin(\vartheta + \varphi)$.

出典：T.S. Blyth and E.F. Robertson, "Basic linear algebra". Springer Undergraduate Mathematics Series. Springer-Verlag 1998.

参考：rotation through ϑ 角 ϑ の回転, frame of reference 座標系, 基準系