A list of errors of our previous paper

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1 Correction

In our previous paper, we give a definition of weak Kleene algebra, called a monodic tree Kleene algebra and claim that monodic tree Kleene algebras are sound and complete for monodic tree languages. However, some errors are included in the paper. Especially, the proof of the completeness theorem contains fatal errors and thus we withdraw the claim.

Error 1. *A monodic tree Kleene algebra does not have right-zero law, i.e.* $a0 = 0$*, but in this setting there is a pair of monodic tree regular expressions* e_1 *and* e_2 *which represent the same language while we cannot prove* $e_1 = e_2$*.*

Explaination Let $e_1 = f(\Box, \Box) \cdot 0$ and $e_2 = 0$. Both e_1 and e_2 represent an empty set but we cannot prove $e_1 = e_2$ without a right-zero law. The reason why we drop a right-zero law is that when we consider general regular tree expression, sometimes a right-zero does not work. For example, for a constant c, we have $c \cdot 0 = c$. $\Box$

The error above means that the completeness theorem is fault and thus we withdraw the completeness theorem.

Error 2. *A monodic tree Kleene algebra has only one-side of left-distribution law, i.e.* $ca + cb \leq c(a + b)$*, but in this setting there is a pair of monodic tree regular expressions* e_1 *and* e_2 *which represent the same language while we cannot prove* $e_1 = e_2$*.*

Explaination Let $e_1 = g(\Box) \cdot (f(\Box, \Box) + h(\Box, \Box))$ and $e_2 = g(\Box) \cdot f(\Box, \Box) + g(\Box) \cdot h(\Box, \Box)$. Both e_1 and e_2 represent the set $\{g(f(\Box, \Box)), g(h(\Box, \Box))\}$ but we cannot prove $e_1 = e_2$ without a both-sided left-distribution law. $\Box$

For avoiding the probelm above, we assume the following restriction of signatures for the definition of monodic regular tree languages.

The arity of a function symbol in Σ is fixed with $n \geq 2$, i.e. $\Sigma = \Sigma_n$.

Error 3. *Matrices of monodic tree Kleene algebras do not hold associativity, i.e.* $(AB)C \neq A(BC)$ *and Lemma 6(ii) is incorrect.*

Explaination Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \qquad C = \begin{pmatrix} i & j \\ k & l \end{pmatrix}.$$

Then we have

$$\begin{aligned} (AB)C &= \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix} \begin{pmatrix} i & j \\ k & l \end{pmatrix} = \begin{pmatrix} aei+bgi+afk+bhk & \cdot \\ \cdot & \cdot \end{pmatrix} \\ A(BC) &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} ei+fk & ej+fl \\ gi+hk & gj+hl \end{pmatrix} = \begin{pmatrix} a(ei+fk)+b(gi+hk) & \cdot \\ \cdot & \cdot \end{pmatrix} \end{aligned}$$

Since a monodic tree Kleene algebra has only one side of distribution law, the 1-1 elements of $(AB)C$ and $A(BC)$ are different. Actually, let us consider two regular tree expressions $a(\square,\square)e(\square,\square)i(\square,\square) + b(\square,\square)g(\square,\square)i(\square,\square) + a(\square,\square)f(\square,\square)k(\square,\square) + b(\square,\square)h(\square,\square)k(\square,\square)$ and $a(\square,\square)(e(\square,\square)i(\square,\square) + f(\square,\square)k(\square,\square)) + b(\square,\square)(g(\square,\square)i(\square,\square) + h(\square,\square)k(\square,\square))$. The first one represents

$$\begin{aligned} &\{a(e(i(\square,\square), i(\square,\square)), e(i(\square,\square), i(\square,\square))), \\ &b(g(i(\square,\square), i(\square,\square)), g(i(\square,\square), i(\square,\square))), \\ &a(f(k(\square,\square), k(\square,\square)), f(k(\square,\square), k(\square,\square))), \\ &b(h(k(\square,\square), k(\square,\square)), h(k(\square,\square), k(\square,\square)))\} \end{aligned}$$

and the second does

$$\begin{aligned} &\{a(e(i(\square,\square), i(\square,\square)), e(i(\square,\square), i(\square,\square))), \\ &a(e(i(\square,\square), i(\square,\square)), f(k(\square,\square), k(\square,\square))), \\ &a(f(k(\square,\square), k(\square,\square)), e(i(\square,\square), i(\square,\square))), \\ &a(f(k(\square,\square), k(\square,\square)), f(k(\square,\square), k(\square,\square))), \\ &b(g(i(\square,\square), i(\square,\square)), g(i(\square,\square), i(\square,\square))), \\ &b(g(i(\square,\square), i(\square,\square)), h(k(\square,\square), k(\square,\square))), \\ &b(h(k(\square,\square), k(\square,\square)), g(i(\square,\square), i(\square,\square))), \\ &b(h(k(\square,\square), k(\square,\square)), h(k(\square,\square), k(\square,\square)))\}. \end{aligned}$$

$\square$

According to the above explanation, we can see that for matrices A, B and C, $(AB)C \le A(BC)$ holds.

Error 4. *The class of monodic tree automata is not closed under determinization, and thus Lemma 11 is incorrect.*

Explaination A monodic tree automaton $\mathcal{A} =$

$$\begin{aligned} \square &\to s \\ \square &\to s' \\ f(s,s) &\to s' \\ f(s',s') &\to s'' \end{aligned}$$

with $Q_f = \{s''\}$ accepts $f(\Box, f(\Box, \Box))$. Assume that there exists a deterministic monodic tree automaton $\mathcal{A}'$ such that $L(\mathcal{A}) = L(\mathcal{A}')$. Since $\mathcal{A}'$ is also monodic, the accepting sequence of $\mathcal{A}'$ can be written as

$$f(\Box, f(\Box, \Box)) \to_* f(q, q) \to q_f$$

for some states q, q_f. According to the rule $f(q, q) \to q_f$ and $\Box \to q$, we have

$$f(\Box, f(\Box, \Box)) \to f(q, q_f)$$

which means that $q = q_f$ according to the determinisity of $\mathcal{A}'$. Then $\Box \in L(\mathcal{A}')$, a contradiction. $\Box$